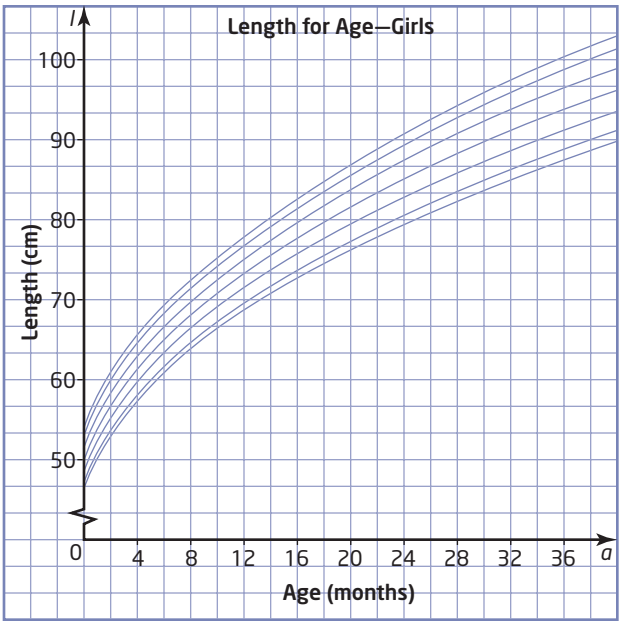
2.5 Linear and Non-Linear Relations



These graphs are called growth charts. Doctors use them to help judge the health of young children. The graphs show the normal range of lengths for boys and girls from birth to age 3.

Investigate

How do linear and non-linear graphs compare?

Use the growth charts to answer the following questions.

1. What is the same about the growth of young girls and the growth of young boys?
2. How does the growth of young girls differ from that of young boys?
3. A graph of a **linear relation** forms a straight line. For what age range does the growth of girls and boys appear to be linear?
4. For what age range does the growth of girls and boys appear to be non-linear?
5. **Reflect** How do changes in non-linear graphs differ from changes in linear graphs?

linear relation

- a relation between two variables that forms a straight line when graphed

line of best fit

- a straight line that comes closest to the points on a scatter plot

curve of best fit

- a curve that comes closest to the points on a scatter plot of a non-linear relation

Often, you can use a linear relation to model the data on a scatter plot. This linear relation corresponds to a **line of best fit**. Lines of best fit pass through or close to as many points as possible. Any points that are not on the line of best fit should be distributed evenly above and below it. A line of best fit can help you make interpolations and extrapolations.

Many non-linear relations can be modelled with a **curve of best fit**. You can draw curves of best fit using the same method as for a line of best fit. A curve of best fit should pass through or close to as many points as possible, and any points that are not on the curve should be distributed evenly above and below it.

Example 1 Use a Line of Best Fit

The gymnophthalmid lizard lives in the Amazon rainforest. Recent research found that this lizard keeps its body temperature close to the temperature of its surroundings. The table lists data from this research.

Surrounding Temperature (°C)	25.0	24.8	27.9	30.3	28.2	24.8	25.6	29.9	25.5	28.4	28.5	28.0	27.9
Lizard's Body Temperature (°C)	26.2	28.2	29.7	30.3	29.8	28.3	27.6	30.8	29.5	30.0	28.8	28.7	29.0

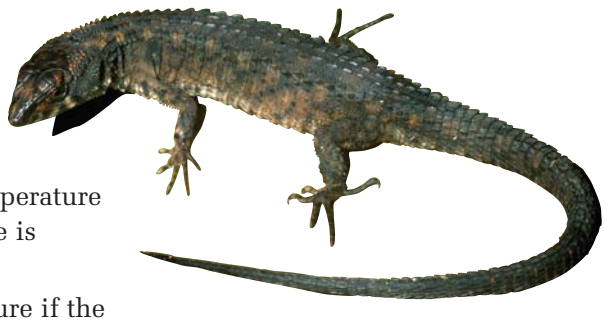
a) Graph the data.

b) How are the two variables related? Is this relationship linear or non-linear? Explain.

c) Draw a line of best fit.

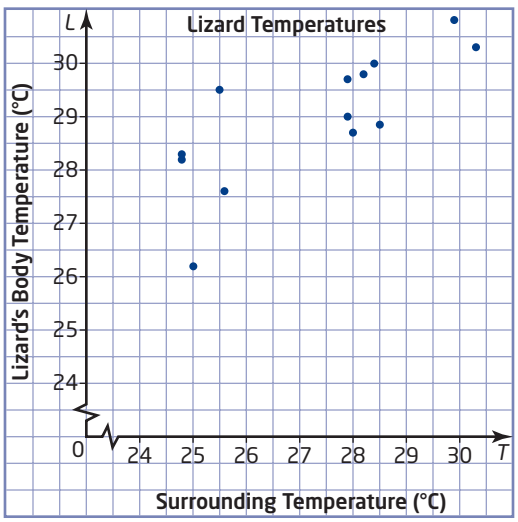
d) Estimate the lizard's body temperature if the surrounding temperature is 26°C.

e) Estimate the lizard's temperature if the surrounding temperature is 35°C.



Solution

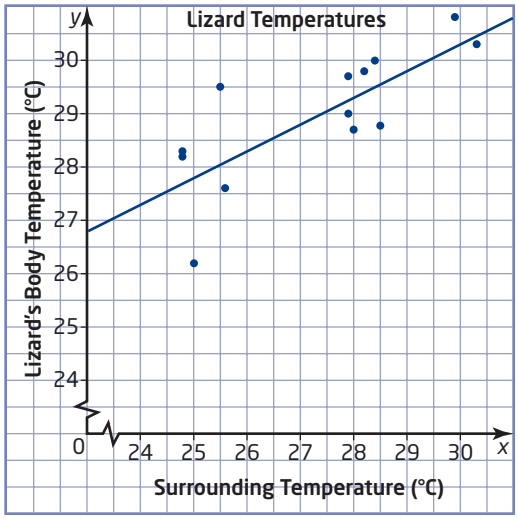
a) Plot the data using a scatter plot.



b) The data points show a clear trend. As the surrounding temperature increases, so does the lizard's body temperature. You can classify the relationship as linear since the data points lie close to a straight line. Although the relationship is not perfectly linear, a linear relation is a good model for the data.

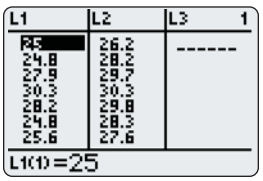
c) Method 1: Use Pencil and Paper

Draw a straight line as close as possible to the points on the graph.



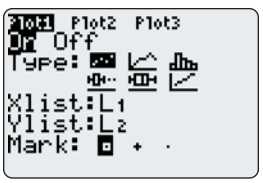
Method 2: Use a Graphing Calculator

Clear all the calculator's lists.
Press STAT and select **1:Edit**.
Enter the surrounding temperatures in list **L1** and the lizard's body temperatures in list **L2**.



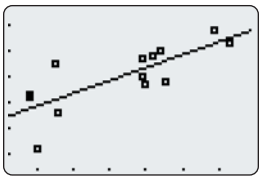
Display the scatter plot:

- Press 2nd [STAT PLOT] to display the **STAT PLOTS** menu.
- Select **1:Plot1** to display the settings for Plot1.
- Select **On** and the scatter plot symbol if they are not already highlighted.
- Make sure that **Xlist** is set to **L1** and **Ylist** is set to **L2**.
- Press ZOOM and select **9:ZoomStat**.



Add the line of best fit:

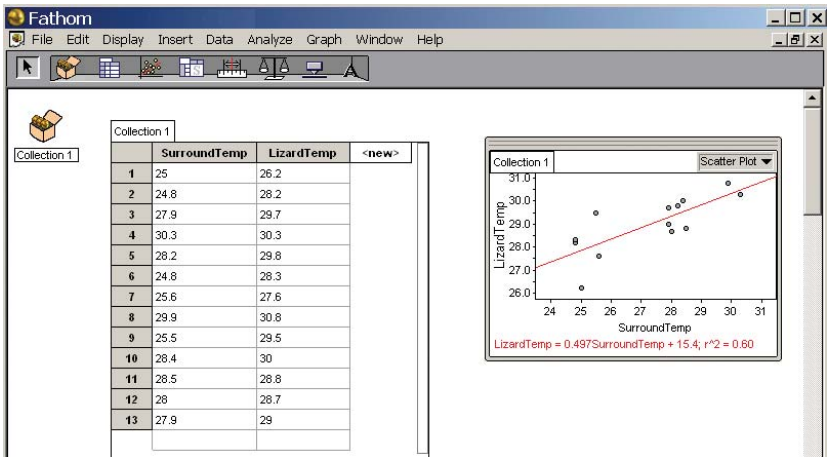
- Press STAT, cursor over to display the **CALC** menu, and select **4:LinReg(ax + b)**.
- Press VARS, and cursor over to display the **Y-VARS** menu.
- Select **1:FUNCTION**; then, select **1:Y1**.
- Press ENTER to get to the LinReg screen, and press GRAPH.



Method 3: Use *Fathom*™

Drag the case table icon onto the desktop.
Enter the headings "SurroundTemp" and "LizardTemp" at the top of the first two columns.
Enter the data into the case table.

Drag the graph icon onto the desktop.
Drag the "SurroundTemp" heading to the horizontal axis and the "LizardTemp" heading to the vertical axis.
To add a line of best fit, click on **Graph**, and choose **Least Squares Line**.



Method 4: Use a Spreadsheet

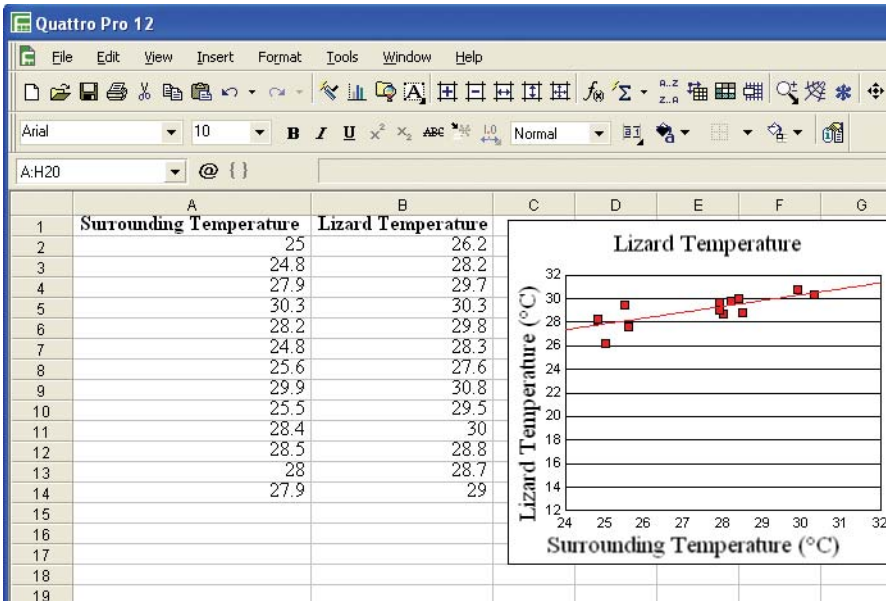
Enter the data in the first two columns of a spreadsheet. Then, select these data.

In Corel® Quattro® Pro, click **Insert/Chart**. Click **Next**.
Uncheck the **3D** box. For chart type, click on **Scatter**, and select the **no line** option. Click **Next** again.
Enter the title for your graph and the labels for the axes.
Click **Finish**.
Then, move the cursor to where you want the graph to appear and click to place it.
To add a line of best fit, select the scatter plot. Right-click on the data points, and select **Series Properties**. Then, click on the **Trendline** tab and select **Linear fit**.

In Microsoft® Excel, click **Insert/Chart**.
Under Chart type, click on **XY (Scatter)**, and click the **Next** button twice.
Enter the title for your graph and the labels for the axes.
Click the **Next** button again; then, click **Finish**.
To add a line of best fit, click on **Chart** and select **Add Trendline**.
In the dialogue box for **Type**, choose **Linear**, and click **OK**.

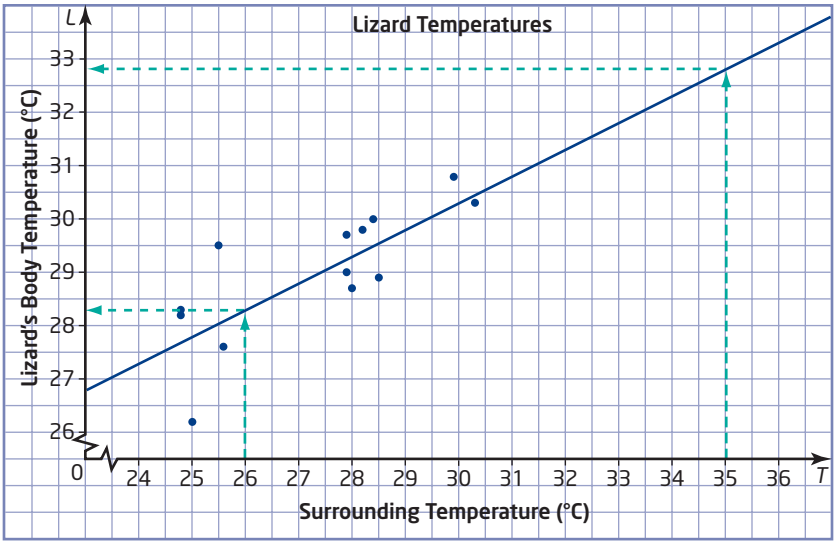
Technology Tip

Ctrl-Z will undo an action in either Corel® Quattro® Pro or Microsoft® Excel. For example, you can use **Ctrl-Z** to restore something you deleted by mistake.



	A	B
1	Surrounding Temperature	Lizard Temperature
2	25	26.2
3	24.8	28.2
4	27.9	29.7
5	30.3	30.3
6	28.2	29.8
7	24.8	28.3
8	25.6	27.6
9	29.9	30.8
10	25.5	29.5
11	28.4	30
12	28.5	28.8
13	28	28.7
14	27.9	29

d) Interpolate using a line of best fit. Read up from 26°C on the horizontal axis to the line of best fit. Then, read across to find that the lizard's body temperature is about 28°C.

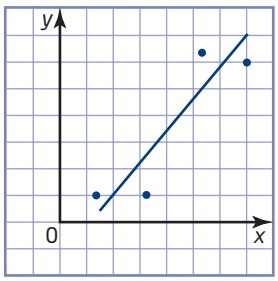


e) Extrapolate using a line of best fit. Extend the line far enough that you can read up to it from 35°C on the horizontal axis. Then, read across to the vertical axis to find the corresponding body temperature of the lizard. The temperature of the lizard will be about 33°C when the surrounding temperature is 35°C. This estimate is reasonable only if the relationship between the two temperatures is still linear at temperatures greater than those in the set of data.

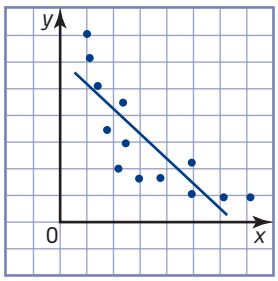
Example 2 Decide Whether a Line of Best Fit Is Appropriate

Why is a line of best fit not a good model for the data in each graph?

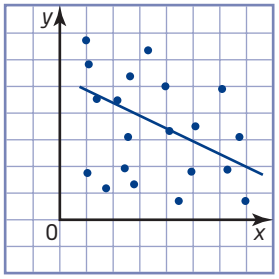
a)



b)



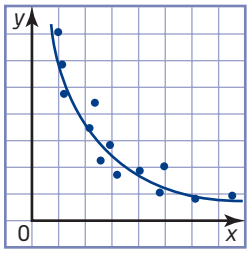
c)



Solution

a) There are not enough data points to determine the relationship between the variables.

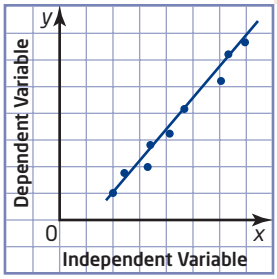
b) The data points tend to follow a curve rather than a straight line. The middle points are all below the line and the points near the ends are mostly above the line. The relation is non-linear. A curve of best fit is a better model for these data.



c) The points have no apparent pattern.

Key Concepts

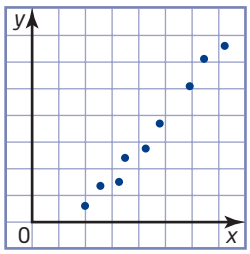
- Data that form a relatively straight line on a scatter plot indicate a linear relationship between the variables.
- A line of best fit can model a linear relationship, but is usually a poor model for a non-linear relationship.
- You can use a line or curve of best fit to interpolate values within a data set.
- You can extrapolate values beyond the range of a set of data by extending a line or curve of best fit.



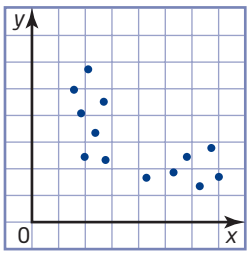
Communicate Your Understanding

C1 Which of these scatter plots shows a linear relationship? Explain.

a)



b)

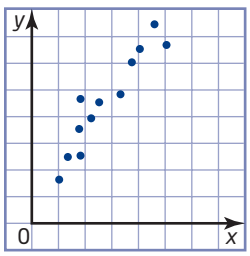


C2 Can you draw a line of best fit that does not pass through any of the data points? Explain your answer. Give an example if possible.

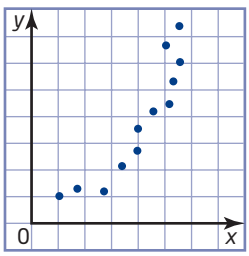
Practise

1. Does each graph show a linear relationship? Explain.

a)



b)



2. Does each set of points have a linear relationship? Justify your answer.

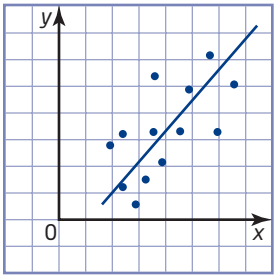
a) $(-6, -4)$, $(-5, -2)$, $(-4, 0)$, $(-3, 2)$, $(-2, 4)$, $(-1, 6)$, $(0, 8)$, $(1, 10)$, $(2, 12)$, $(3, 14)$

b) $(0, 0)$, $(1, 1)$, $(2, 4)$, $(3, 9)$, $(4, 16)$, $(5, 25)$, $(6, 36)$

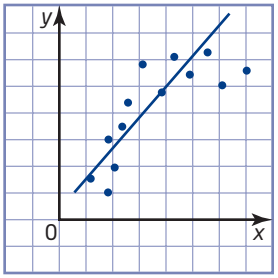
For help with questions 3 and 4, see Example 2.

3. State whether each of these lines of best fit is a good model for the data. Justify your answers.

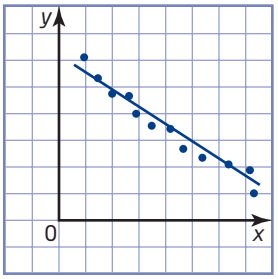
a)



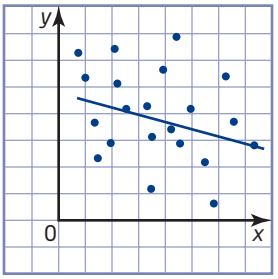
b)



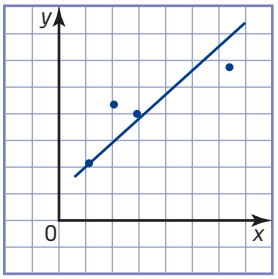
c)



d)



e)



4. Plot each set of points on a grid. If your plot shows a linear relationship, draw a line of best fit. If the relation appears non-linear, sketch a curve of best fit.

a)

x	1	2	7	4	9	3	6	2
y	−2	0	12	5	20	3	11	1

b)

Time (s)	40	32	55	18	66	43	37
Score	7	9.5	6	10	4	6	7.5

c) $(-5, 3)$, $(6, 1)$, $(2, 2)$, $(-3, 0)$, $(-1, 2)$, $(9, 10)$, $(8, 4)$, $(0, 1)$, $(7, 5)$, $(-4, 1)$

Connect and Apply

5. A weather balloon recorded the air temperature at various altitudes.

Altitude (m)	500	800	1000	1500	1700	2100
Temperature (°C)	16.2	14.5	13.1	11.2	9.8	8.1

a) Make a scatter plot of the data.

b) Describe the relation and draw a line or curve of best fit.

c) Use your line or curve of best fit to estimate the temperature at an altitude of 600 m.

d) Estimate the temperature at 2500 m.

Did You Know?

About 1800 weather balloons are launched every day to measure conditions high in the atmosphere.

6. Farmers have found that the spacing between plants affects the crop yield. This table lists data for canola.

Density (plants/m^2)	Plants With Good Yield (%)
20	57.0
40	85.0
60	91.5
80	94.0
100	95.5
120	96.5
140	96.0
160	95.5
180	95.0
200	94.5
300	91.5
400	86.5
500	85.0

a) Make a scatter plot of the data.

b) Describe any trends you see in the scatter plot. What type of relation do these trends indicate?

c) Is a line of best fit a good model for the data? Explain why or why not.

d) Suggest two factors that could affect the relation between planting density and crop yields for canola.

7. a) This table lists the speed of a skydiver during the first 4 s of free fall. Plot the data on a grid with time from 0 s to 12 s on the horizontal axis and speed from 0 m/s to 100 m/s on the vertical axis.

Time (s)	0	1	2	3	4
Speed (m/s)	0	6	12	18	23

b) Extrapolate to estimate the skydiver's speed after 12 s of free fall.

c) This table gives the skydiver's speed for the next 8 s of free fall. Add these data to the graph you made in part a).

Time (s)	5	6	7	8	9	10	11	12
Speed (m/s)	28	33	37	40	42	43	43	43

d) Describe the trend in the enlarged set of data. What causes this trend?

e) Explain why extrapolations can be inaccurate.

Did You Know?

Leonardo da Vinci drew designs for a parachute around 1492. However, the first known parachute jump was made by Faust Vrancic in 1617.

8. Conduct an experiment to investigate how a person's heart rate changes immediately after exercise. Work with a partner.

a) ***Objective:*** Describe the purpose of your experiment.

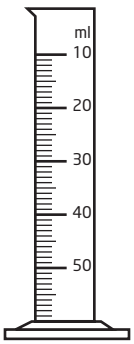
b) ***Hypothesis:*** Make a hypothesis about the trend in a person's heart rate after exercise.

c) ***Procedure:*** One partner runs on the spot for 2 min while the other partner keeps track of the time. Then, the runner counts his or her pulse over 10-s intervals for a total of 80 s. The other partner records the data in a table.

d) ***Observations:*** Graph your measurements. Draw a line or a curve of best fit. Compare your graph with those drawn by your classmates. Do you see any pattern in the way your heart rates slow down?

e) ***Conclusion:*** What inference can you make from the data? Do your observations support your hypothesis?

f) ***Evaluation:*** Did the experiment meet its objective? Could there be any errors in your data? How could you improve the experiment?



9. Design and carry out an experiment to see if there is a linear relation between the height of water in a graduated cylinder and the number of pennies dropped into the water. Write a report on your experiment. This report should include

a) the objective of the experiment

b) your hypothesis

c) a description of your procedure

d) your observations

e) your conclusions

f) an evaluation of the experiment

Achievement Check

10. This table shows a series of measurements of water temperature at various depths below a research ship.

Depth (m)	100	200	300	400	500	800	1000	1200
Temperature (°C)	19.4	19.0	18.1	17.5	16.0	9.7	6.2	6.0

a) Make a scatter plot of the data.

b) Draw a line or curve of best fit.

c) Describe the relationship between the variables.

d) Estimate the water temperature at a depth of 700 m.

e) Extrapolate to estimate the temperature at a depth of 1600 m.

f) Which of your two estimates is likely to be more accurate? Explain your reasoning.

Extend

11. Consider each set of data. How can you tell whether the relation between the variables in each pair is linear without graphing the data?

a)

t	−2	−1	0	1	2	3	4	5
d	−9	−4	1	6	11	16	21	26

b)

t	−3	−2	−1	0	1	2
h	5	0	−3	−4	−3	0

12. Gayle recorded the distances she drove and the readings of the fuel gauge in her truck.

Distance Travelled (km)	Fuel Gauge Reading (eighths)
0	8 (full)
105	7
205	6
300	5
395	4
460	3
525	2
580	1
625	0

If the truck's fuel efficiency was constant, what can you conclude about the relation between the fuel gauge reading and the amount of fuel left in the tank? Explain your reasoning.

13. **Math Contest** If n is positive, which of the following expressions always has a value less than 1?

A $\frac{1}{n}$ **B** $\frac{1-n}{n}$

C $\frac{1+n}{n}$ **D** $\frac{n}{n+1}$

14. **Math Contest** Determine the number of even three-digit numbers that can be made by choosing from the digits 1, 2, 3, 4, 5, and 6 if each number must contain three different digits.